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JUMPS WITH RADIATION IN MODELS DESCRIBED BY THE GENERALIZED KORTEWEG–DE VRIES EQUATION†

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One of the main types of jumps in reversible systems – jumps with radiation – is investigated using the example of the generalized Korteweg–de Vries equation. A method of obtaining the relations at the jump without analysing its structure is indicated. Averaged equations and equations for the centred simple envelope wave which give a fairly simple and effective description of the jump in the non-local sense are derived. A method for the numerical solution of these equations is described. Methods of investigating the structure of the jump are analysed. One of the methods consists of the fact that the structure of the jump with radiation is regarded as part of a fundamental type of solitary wave – a multi-soliton with an infinite number of elementary solitary waves.

The general theory of jumps in reversible systems was described previously in [1] and was applied specifically to the example of generalized Korteweg–de Vries equations in [2].

1. INTRODUCTION

When solving the problem of the decay of an arbitrary discontinuity in models without dissipation, in the general case wave zones arise which spread out with time and are described by certain averaged equations. At the boundaries of these zones or inside them there may be localized (unexpanding) transitions between uniform, periodic and quasi-periodic states. These transitions are regarded as jumps. One of these – the transition between a uniform and a periodic state – has been called a jump with radiation. This type of jump is shown in Fig. 1(a). The region to the right of the jump will be referred to the region in front of the jump, and the region to the left will be referred to the region behind the jump; these will be denoted by the numbers 1 and 2 respectively. Note that we may also mean by a jump (in the non-local sense) the transition between two uniform states, separated by an expanding wave zone [3].

The conditions for a jump with radiation to exist were formulated in [1]. Suppose U is the velocity of the jump and $\omega = \omega(k)$ is the dispersion relation for the linearized model. It is required that, for the state behind the jump, the straight line $U = \omega/k$ should not intersect the dispersion curve, except when $k = 0$. However, for the state in front of the jump there must be one intersection. These conditions also simultaneously ensure the evolution of the jump, i.e. stability, and, of course, its observability in a numerical experiment as well. We also introduce, as an additional condition for existence, the assumption that, for the state behind the jump, the solutions of the equation $U = \omega/(k)/k$ must be complex. The correctness of these assertions were confirmed by numerical experiments with the generalized Schrödinger equation [4], the Korteweg–de Vries equation [2], the Boussinesq equation [5], and also in a cold plasma and a plasma with hot electrons [6].

The generalized Korteweg–de Vries equation is the simplest equation to analyse in which a jump with radiation is encountered. It describes the propagation of waves along the surface of a liquid with an ice or some other coating [7]. We will present this equation in the form of a conservation law (from the physical point of view this is the law of conservation of momentum)

$$a_t + (a^2/2 + b_3 a_{xx} + b_5 a_{xxxx})_x = 0 \tag{1.1}$$

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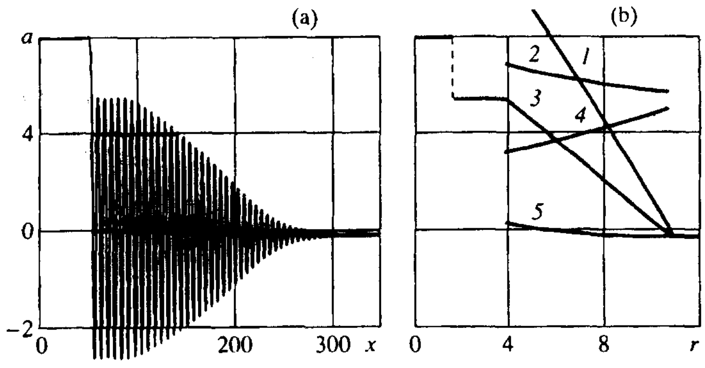

Fig. 1

The law of conservation of energy in the linear approximation has the form

$$(a^2/2)_t + [a^3/3 + b_3(aa_{xx} - a_x^2/2) + b_5(aa_{xxxx} - a_x a_{xxx} + a_{xx}^2/2)]_x = 0 \tag{1.2}$$

There is one other conservation law in this model, namely, the non-linear correction to the law of conservation of energy [2]. However, it has been found that when obtaining the relations at the jump (see below in Section 2) this conservation law turns out to depend on the first two, and hence it has not been used. Hence, the methods employed below should be regarded as standard for may models, since in typical complete non-asymptotic models without dissipation, there is precisely one additional physical conservation law – the law of conservation of energy.

The results of dynamic (non-stationary) calculations of the jumps for the equation considered were presented in [2, 5]. It is convenient to investigate the initial equation in the normalized form: $b_3 = \pm 1$ and $b_5 = 1$. This normalization corresponds to the case of waves on the surface of a liquid with an ice cover [5]. Here $\Delta = a_2 - a_1$ is the amplitude of the jump in the non-local sense, and U is the velocity of the local jump (here and below we have in mind the values of k and ω for the region behind the jump). The form of the solutions with a jump with radiation when $b_3 = -1$ and $b_3 = 1$ is qualitatively the same; it is shown in Fig. 1(a). When $b_3 = -1$ a jump with radiation occurs if $\Delta > \Delta_{*-} \approx 0.4$. As the theory also predicts, this amplitude corresponds to a transition from real values of k to complex values. When $b_3 = 1$, a jump with radiation occurs if $\Delta > \Delta_{*+} \approx 0.69$. The theory also predicts a range of values of $\Delta_1 < \Delta < \Delta_2$, in which both a jump with a stationary structure and a non-stationary structure can occur, i.e. it does not give an exact critical value; $\Delta_1 < \Delta_{*+} < \Delta_2$, the value of Δ_1 corresponds to the case $U = \partial^2\omega/\partial k^2$ when $k > 0$. The value of Δ_2 corresponds to the case $U = \max_k \partial^2\omega/\partial k^2$. The factors which determine the values of Δ_{*+} are discussed in Section 6.

In this paper we consider methods of investigating a jump, based on an analysis of the stationary solutions of Eq. (1.1). Suppose $(u_0, u_1, u_2, u_3) \equiv (a, a_x, a_{xx}, a_{xxx})$. We will assume that the numerical solutions of the system of equations describing travelling waves

$$u_0' = u_1, \quad u_1' = u_2, \quad u_2' = u_3, \quad u_3' = (P - b_3 u_2 - u_0^2/2 + V u_0)/b_5 \tag{1.3}$$

are accessible for analysis, where P is the constant of integration and V is the phase velocity. In Section 2 we describe a method of obtaining the parameters of the periodic state in front of the jump without finding the phase trajectory which describes its structure. In Section 3 we derive the averaged equations for the envelope of the wave zone. In Section 4 we describe a method of calculating centred simple waves. The results of Section 2–4 enable us to give a complete quantitative description of a jump in the non-local sense. In Section 5 we give the simplest method of finding the structure of a local jump with radiation, i.e. the solution which describes the transition from the uniform to the periodic state. In Section 6 we describe a systematic approach to finding the structure which enables us to relate these jumps to soliton-type solutions, we prove their existence and we investigate all possible solutions.

2. THE PERIODIC STATE IN FRONT OF THE JUMP

We will obtain initial data for the solution of system of (1.3), corresponding to the hump on the graph $a(x)$ for the periodic state in front of the jump. This action is equivalent to finding two additional relations on the jump [1], since in this case we find the values of a and a_{xx}.

We have $a_x = a_{xxx} = 0$. We obtain the following relations from the momentum and energy integrals

$$f(a_{1*},U)+b_3a_{xx1*}+b_5a_{xxxx1*}=f(a_2,U),\quad f\equiv -Ua+a^2/2$$

$$F(a_{1*},U)+b_3a_{1*}a_{xx1*}+b_5a_{1*}a_{xxxx1*}+b_5a_{xx1*}^2/2=F(a_2,U),\quad F\equiv -Ua^2/2+a^3/3$$

$$a_{xx1*}=-\{2[F(a_2,U)-F(a_{1*},U)-a_{1*}(f(a_2,U)-f(a_{1*},U))]/b_5\}^{1/2}$$

where U is the velocity of the jump, the subscripts 1 and 2 denote states in front of the jump and behind it, while the asterisk indicates that we have in mind a periodic state of the local jump and not a uniform non-local state; $P = f(a_2, U)$, $V = U$.

The method of finding a periodic solution in front of the jump consists of the following. The initial conditions are specified: $a = a_0$ (the unknown variable quantity), $a_x = 0$, a_{xx} – from the above formula and $a_{xxx} = 0$. For any a_0 in a certain range of values we can find point x_1 such that $a_x(x_1) = 0$. We then choose the value $a = a_{1*}$ such that $a_{xxx}(x_1) = 0$. The section $[0, x_1]$ comprises the half-period, and the solution in the second half-period can be obtained by symmetrization.

In the case when a dynamic calculation shows the presence of a jump with radiation, this periodic solution is in fact obtained for the specified values of a_2 and U. Note that this periodic solution is also obtained when there is no jump with radiation in the dynamic calculation, and therefore an analysis of the existence and stability of the structure of the jump is necessary.

Hence, as in the case of a kink-type jump [2], one additional conservation law turns out to be sufficient to enable the necessary relations on the jump with radiation to be obtained without analysing its structure.

3. AVERAGED EQUATIONS

We have the momentum and energy integrals

$$-Va+a^2/2+b_3a_{xx}+b_5a_{xxxx}=P \tag{3.1}$$

$$-Va^2/2+a^3/3+b_3(aa_{xx}-a_x^2/2)+b_5(aa_{xxxx}-a_xa_{xxx}+a_{xx}^2/2)=E \tag{3.2}$$

The momentum integral is the basis of Eqs (1.3). We will consider all possible periodic solutions of Eqs (1.3).

The method of finding periodic solutions in the general case is as follows. We are given the initial conditions: $u_0(0) = z$, $u_1(0) = 0$, $u_3(0) = 0$. By varying the parameter $u_2(0)$ we obtain the trajectory $\mathbf{u}(x)$ such that a value of x_l exists so that $u_1(x_l) = u_3(x_l) = 0$ (we initially find x_l: $u_1(x_l) = 0$, and we then choose $u_2(0)$: $u_3(x_l) = 0$). The section $[0, x_l]$ is the half-period, and the solution in the second half-period can be obtained by symmetrization.

Remark. In Sections 2 and 3 it is assumed that when $x > 0$ in the section $[0, x_l]$ there is only one value of x: $u_1 = 0$, i.e. the calculation using Eqs (1.3) ceases when this value is reached. This eliminates from consideration doubly periodic solutions and branching of the trajectories, which occurs in the investigations presented in Section 5 and 6, where the calculation is completed in order to avoid overfilling, if $|u_0| > M$, where M is a certain large quantity.

Suppose $P = f(a_2, U) =$ const. We multiply the periodic solutions obtained by the substitution $a \to a + \delta$. We make the same substitution in relations (1.1) and (1.2) and average the equations obtained over a period. We obtain

$$\langle a+\delta\rangle_t+\langle \delta^2/2+\delta a+Va\rangle_x=0$$

$$\langle (a^2+2\delta a+\delta^2)/2\rangle_t+\langle \delta^3/3+\delta^2a+\delta a^2+\delta(P+Va-a^2/2)+Va^2/2+E\rangle_x=0$$

$$k_t+[k(V+\delta)]_x=0\quad (k_t+\omega_x=0)$$

(the angle brackets denote the operation of averaging). To eliminate higher derivatives from the first two equations we used integrals (3.1) and (3.2). The equation are written in a compact form and it is understood that $\langle\delta a\rangle = \delta\langle a\rangle$, $\langle\delta a^2\rangle = \langle\delta a^2\rangle$, etc. The last of the equations of this system is the equation of compatibility [8], and its more usual form is written in the angle brackets; k is a quantity inversely proportional to the wavelength. The independent unknowns are $V, \delta, z = u_0(0)$. The dependent calculated quantities, which occur in the averaged equations are

$$k = x_l^{-1},\ \langle a\rangle = \int_0^{x_l} u_0(x)dx / x_l,\ \langle a^2\rangle = \int_0^{x_l} u_0^2(x)dx / x_l \tag{3.3}$$

$$E = -Vz^2/2 + z^3/3 + z(P + Vz - z^2/2) + b_5 u_2(0)^2/2$$

The physically significant quantities are: $z + \delta$ is the upper envelope, $V + \delta$ is the phase velocity, $\langle a\rangle + \delta$ is the mean flow, $2x_l$ is the wavelength and $z - a(x_l)$ is the amplitude of the oscillations.

The system of averaged equations has a fairly simple form, due to the fact that it is not required to carry out averaging over the higher derivatives; moreover, the quantities (3.3) depend solely on two unknowns: V and z.

4. CENTRED SIMPLE WAVES

We will assume that all the unknowns depend solely on one variable $r = x/t$, and we obtain equations for centred simple waves

$$r\frac{d}{dr}\frac{\langle a^2\rangle + 2\delta\langle a\rangle + \delta^2}{2} - \frac{d}{dr}\left[\frac{\delta^3}{3} + \delta^2\langle a\rangle + \delta\langle a^2\rangle + \delta\left(P + V\langle a\rangle - \frac{\langle a^2\rangle}{2}\right) + V\frac{\langle a^2\rangle}{2} + E\right] = 0$$

$$r\frac{d}{dr}(\langle a\rangle + \delta) - \frac{d}{dr}\left(\frac{\delta^2}{2} + \delta\langle a\rangle + V\langle a\rangle\right) = 0,\quad r\frac{d}{dr} - \frac{d}{dr}[(V+\delta)k] = 0$$

We convert them to the form

$$D\frac{d}{dr}(\delta, V, z)^\tau = 0$$

$$d_{11} = r(\langle a\rangle + \delta) - (\delta^2 + 2\delta\langle a\rangle + P + V\langle a\rangle - \langle a^2\rangle/2)$$

$$d_{21} = r - (\langle a\rangle + \delta),\quad d_{31} = -k$$

$$d_{12} = r(\langle a^2\rangle_v/2 + \delta\langle a\rangle_v) - [\delta^2\langle a\rangle_v + \delta\langle a^2\rangle_v + \delta(\langle a\rangle + V\langle a\rangle_v - \langle a^2\rangle_v/2) + \langle a^2\rangle/2 +$$
$$+ V\langle a^2\rangle_v/2 + E_v]$$

$$d_{22} = r\langle a\rangle_v - (\delta\langle a\rangle_v + \langle a\rangle + V\langle a\rangle_v),\quad d_{32} = rk_v - [k + (V+\delta)k_v]$$

$$d_{13} = r(\langle a^2\rangle_z/2 + \delta\langle a\rangle_z) - [\delta^2\langle a\rangle_z + \delta\langle a^2\rangle_z + \delta(V\langle a\rangle_z + \langle a^2\rangle_z +$$

$$+\delta(V\langle a\rangle_z - \langle a^2\rangle_z/2) + V\langle a^2\rangle_z/2 + E_z]$$

$$d_{23} = r\langle a\rangle_z - (\delta\langle a\rangle_z + V\langle a\rangle_z),\quad d_{33} = rk_z - (V+\delta)k_z$$

We obtain the characteristic velocities c_i in front of the jump and the value $r = r_0$ at the beginning of the centred simple wave, and we express δ in terms of the remaining unknowns and the variable r

$$\det D = b_{r0} + b_{r1}r + b_{r2}r^2 + b_{r3}r^3 = b_{\delta 0} + b_{\delta 1}\delta + b_{\delta 2}\delta^2 + b_{\delta 3}\delta^3$$

$$c_i = R_{3i}(b_{r0}, b_{r1}, b_{r2}, b_{r3})$$

$$c_1 < c_2 = r_0 < c_3,\quad \delta = R_{3j}(b_{\delta 0}, b_{\delta 1}, b_{\delta 2}, b_{\delta 3})$$

where R_3 is the root of the cubic polynomial from Cardano's formula. We choose as the value of δ the one of the three roots for which $\delta(r_0) = 0$. A knowledge of the characteristic velocities enables us to verify formally that the evolution conditions at the jump with radiation is satisfied [1]: $c_1 < U < c_2 < c_3$ in front of the jump and $U < c = a_2$ behind the jump. It is obvious that, in more complex models, the order of the polynomial will be higher. For the generalized Boussinesq equations it will be of the fourth order, i.e. the roots are still found analytically. For more complex models of the plasma type [6] the roots have to be found numerically. This is a unique feature, specific to this model.

We will get rid of the unknown δ. We have

$$\begin{Vmatrix} d_{21}\delta_v + d_{22} & d_{21}\delta_z + d_{23} \\ d_{31}\delta_v + d_{32} & d_{31}\delta_x + d_{33} \end{Vmatrix} \begin{Vmatrix} \dfrac{d}{dr} \end{Vmatrix} \begin{Vmatrix} V \\ z \end{Vmatrix} = -\delta_r \begin{Vmatrix} d_{21} \\ d_{31} \end{Vmatrix}, \quad \delta_r = 1$$

The system is integrated numerically from $r = r_0$ as long as $z \neq \langle a \rangle$; the initial conditions are: $V(r_0) = U, z(r_0) = a_{1*}$.

An example of the calculation of a non-local jump using the average equations and a comparison with the dynamic calculation are presented in Fig. 1. A graph of the solution with a jump with radiation for the non-stationary calculation (a) and graphs of the characteristic physical quantities, obtained using the averaged equations (b) are shown: 1 is the amplitude of the oscillations, 2 is the wavelength of the half-wave, 3 is the upper envelope, 4 is the phase velocity, and 5 is the mean flow (also shown in Fig. 1(a) by the heavy curve); $b_3 = -1$ and $b_5 = 1$.

5. STRUCTURE OF THE JUMP

On the cyclic trajectory $\mathbf{u}_*$, obtained above in Section 2, compatible with the jump, we take the initial point $Q(a_{1*}, 0, a_{xx1*}, 0)$. We linearize Eq. (1.1) in the neighbourhood of the state behind the jump $a = a_2 + a'$, and we obtain waves which grow as $x \to +\infty$ [1]. Substituting $a' \sim \exp\varkappa x$, we obtain

$$-U + a_2 + b_3\varkappa^2 + b_5\varkappa^4 = 0$$

$$\varkappa = [(-b_3 + [b_3^2 - 4(a_2 - U)b_5]^{1/2})/(2b_5)]^{1/2}, \quad \mathrm{Re}(\varkappa) > 0$$

Here we mean the case when $\mathrm{Im}(\varkappa) \neq 0$.

Consider a model of a two-dimensional phase subspace S_2 of trajectories which, as $x \to -\infty$, tend to the uniform state $(a_2, 0, 0, 0)$ – trajectories with initial data

$$\begin{aligned} &u_0 = a_2 + \varepsilon\sin\phi, \quad u_1 = \varepsilon(R\sin\phi + I\cos\phi) \\ &u_2 = \varepsilon(R^2\sin\phi + 2RI\cos\phi - I^2\sin\phi) \\ &u_3 = \varepsilon(R^3\sin\phi + 3R^2I\cos\phi - 3RI^2\sin\phi - I^3\cos\phi) \\ &R = \mathrm{Re}(\varkappa), \quad I = \mathrm{Im}(\varkappa) \end{aligned} \tag{5.1}$$

The trajectory corresponding to the structure of the jump satisfies the condition

$$\lim_{\varepsilon\to 0}\min_{\phi\in[0,2\pi]}\min_x \| \mathbf{u}(x) - Q \|_3 = 0$$

$$\| \ \|_3 = | u_0 - a_{1*} | + | u_1 | + | u_3 |$$

In Fig. 2 we show an example of the numerical solution ($b_3 = 1, b_5 = 1$), chosen as the phase trajectory that is the minimum distance from the point Q in accordance with the above rule. The number of periods is limited by the computer accuracy.

6. A SYSTEMATIC APPROACH TO FINDING THE STRUCTURE

A jump is regarded as the limit of a sequence of solutions describing a series of solitary waves, obtained by symmetrization about a point on any hump or trough for the periodic state in front of a jump. We will investigate all possible solitary waves by investigating the intersections of the subspace S_2 with the subspace S: $u_1 = 0$ and $u_3 = 0$, i.e. using the criterion of the existence of solitary waves, formulated in [1].

The method of obtaining a solution of the solitary-wave type is as follows. We take the initial data (5.1) and a fairly small value of ε. By varying ϕ we find the trajectory in the section $[0, x_l] : u_1(x_l) = 0$,

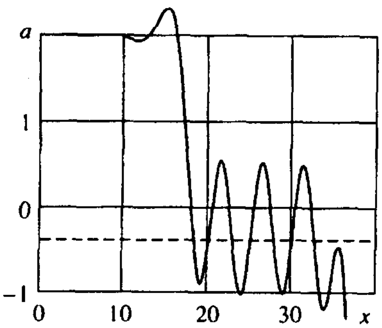

Fig. 2

$u_3(x_l) = 0$, then in the section $[x_l, 2x_l]$ the trajectory is found by symmetrization, and for $x < 0$ and $x > 2x_l$ the solution is proportional to $\exp \varkappa x$ and $\exp -\varkappa x$, respectively.

By virtue of the energy integral, u_2 is expressed in terms of u_0, u_1 and u_3, and hence, in fact, the whole investigation can be carried out in three-dimensional phase space (u_0, u_1, u_3). It is more convenient to investigate the intersection of S_2 (a two-dimensional surface in three-dimensional space) with the plane S: $u_1 = 0$. The intersection represents a certain curve in the (u_0, u_3) plane. A solitary wave corresponds to each point of intersection of this curve with the u_0 axis. To model this curve, for the initial data (5.1), ϕ is varied with a certain step $\Delta\phi$. If, for a certain value of x, $u_1 = 0$, then the corresponding values of u_0 and u_3 are written in a certain file, which is later interpreted by graphical methods as a certain set of points in the (u_0, u_3) plane.

The case $b_3 > 0$, $b_5 > 0$ is of greatest interest since here there is a range of values $\Delta_1 < \Delta = a_2 - a_1 < \Delta_2$ in which both a jump with a stationary structure and with a non-stationary structure can occur (see the introduction).

In Fig. 3 we show a section, and Fig. 4 the corresponding solitary waves, for the case when the amplitude of the jump is fairly large, i.e. it is known from a dynamic calculation that the structure of the jump exists. The section represents a certain curve in S space with a denumerable number of branches. In view of the very complex structure it is shown in Fig. 4 in the form of individual points (and not connected curves), from which one can easily construct certain branches visually. Only branches corresponding to the simplest types of trajectories can be seen. When the accuracy of the calculation is increased and the step $\Delta\phi$ used to construct the figure of the cross-section is reduced, the density of the points on the branches, the number of branches observed and, correspondingly, the number of solitary waves, can be increased. Here we can only see nine intersections, to which solitary waves of different types correspond. In Figs 3 and 4 the letters (a)–(i) are ascribed to these intersections, and the intersection with $u_0 = 2$ is not taken into consideration since there is an equilibrium point there. The value $u_0 \equiv V \equiv a_{1e} = 2U - a_2$, corresponding to the other equilibrium point (1.3), i.e. the equilibrium point for the region in front of the jump, is denoted in Figs 2–6 by the dashed line. The equilibrium point corresponding to the state behind the jump can be clearly seen in Fig. 3 and especially in the analogous Fig. 5, since a spiral branch curls round this point.

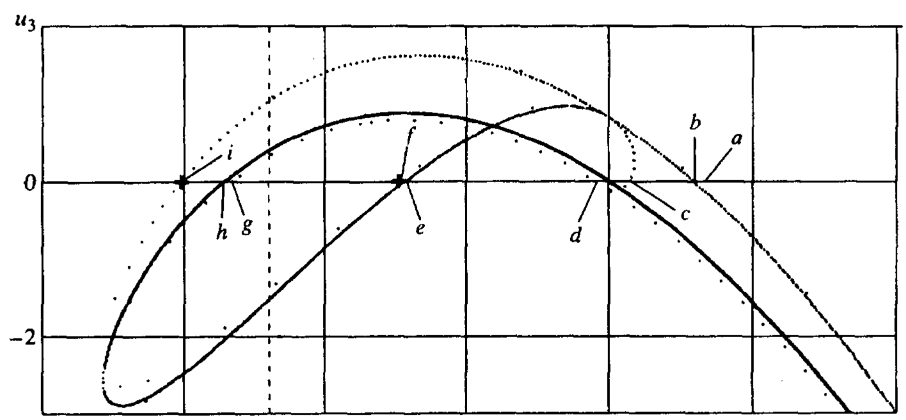

Fig. 3

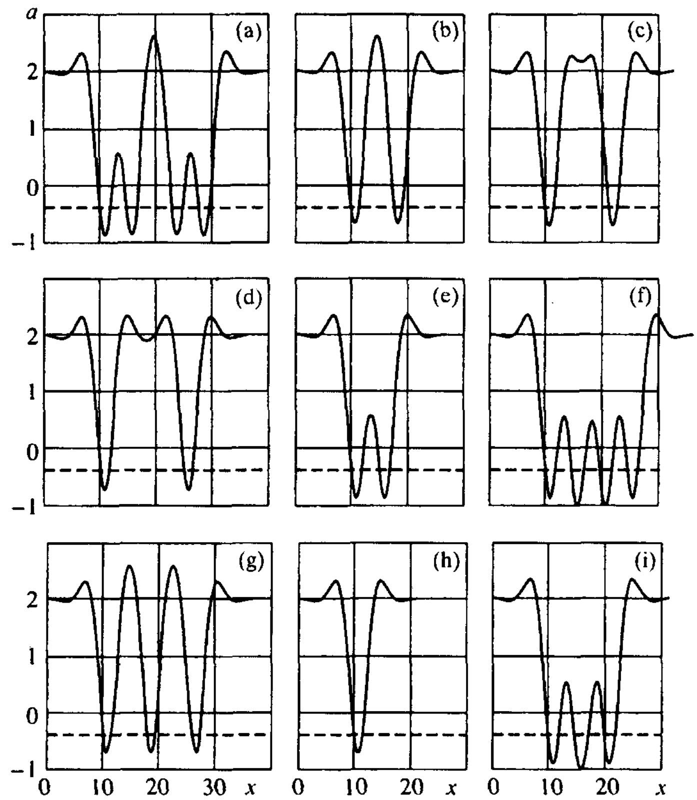

Fig. 4

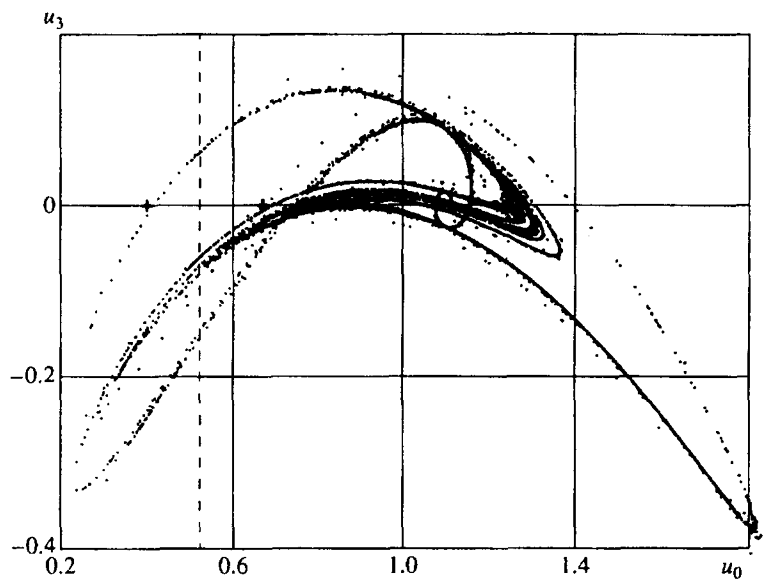

Fig. 5

It is obvious that the wave shown in Fig. 4(h) is essentially a solitary wave, and all the remaining ones are combinations of local solitary waves, fixed distances from one another, i.e. multi-solitons. By two bold crosses in Figs 3 and 5 we show the points corresponding to the hump and the trough for the periodic state in front of the jump. The branches are condensed in the region of these points. Correspondingly, the solitary waves *e*, *i* and *f* form the beginning of a train of solitary waves, converging to the solution, which can be interpreted as a combination of an evolution jump with radiation and a non-evolution jump inverse to it.

The following empirically established fact should be noted: if the phase trajectory makes a few rotations around the lower equilibrium point and passes close to points corresponding to a hump and a trough for the state in front of the jump, the number of these revolutions can be arbitrarily increased by choosing the value of ϕ, all being limited solely by the computer accuracy, since if $|\phi_1 - \phi_2|$ is less than a certain small quantity, the trajectory may turn out to be the same, in view of computer rounding.

In Fig. 5 we show a section for the case when the dynamic calculation reveals that there is no solution with a jump with a stationary structure, but the theory [1] allows of the existence of such a jump. Here the number of branches is very large and it is not possible to investigate all the visible intersections. However, it can be clearly seen that branches go to points corresponding to a hump and a trough for the periodic state, the method of finding which is described in Section 2, that indicates that the structure of the jump exists. In Fig. 6 we show a few multi-solitons of jump-shaped form (a–c), strictly a solitary wave, the so-called 1:1 soliton-trough (d), a 1:1 soliton-hump (e) [9] and the results of a dynamic calculation for comparison (f).

The solution corresponding to the structure of a jump is obviously not unique, and this also holds for the previous case. However, there is no problem in selecting the solution. For a dynamic calculation with initial data of the tanh type one selects the solution with the simplest structure, and the remaining ones can obviously be observed only if a special choice is made of the initial data with oscillatory-type functions.

Comparing Figs 4 and 6 we can conclude that the main difference between the case of jumps with a moderate amplitude and the case of jumps with a fairly large amplitude is the fact that the 1:1 soliton as a whole is higher than the lowest equilibrium point (1.3). At the same time, there is a periodic part in the multi-solitons, generated by this equilibrium position. Moreover, the structure of a jump contains non-monotonically varying oscillations as x changes, where the position of the local maxima and minima are such that $a_{\max} > a_2$ and $a_{\min} > a_{1e}$, i.e. at least one solitary wave, containing a structure, lies as a whole, above the lowest equilibrium position. We can assume that this formation from solitary waves is unstable. In a dynamic calculation only structures with monotonically decreasing oscillations behind the jump are encountered.

An example of the graph of a dynamic calculation corresponding to this case is shown in Fig. 6(f), which is drawn to the same scale as the solitary waves. In Fig. 7 we show a graph, to a finer scale with

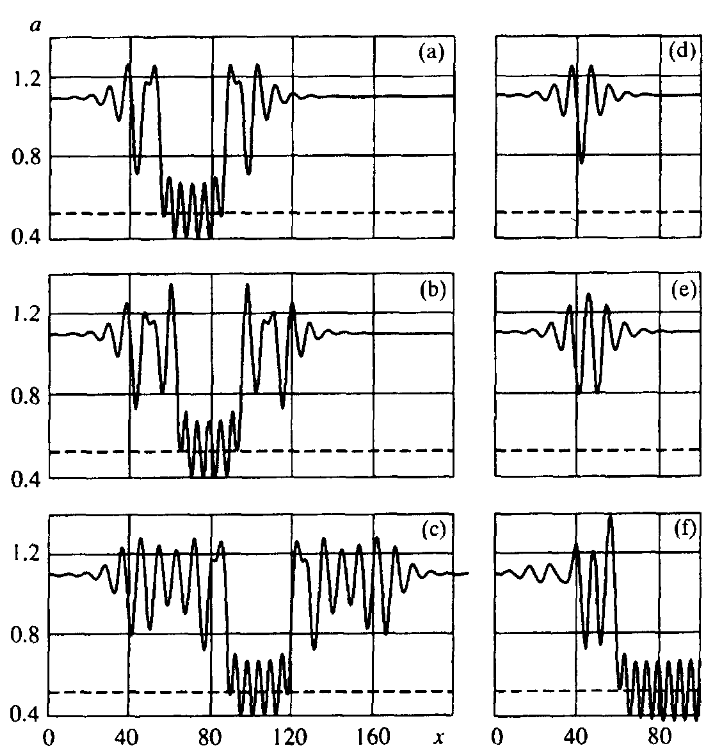

Fig. 6

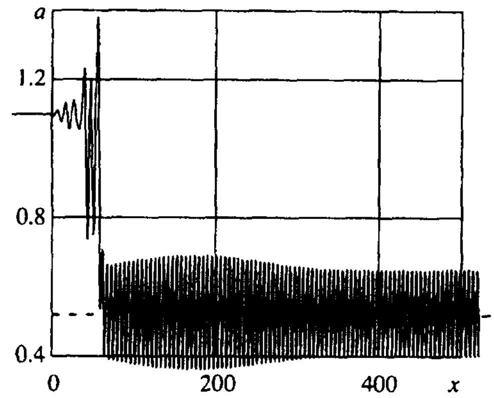

Fig. 7

respect to x, which enables us to show how the radiation of a wave at different instants of time occurs: initially one observes the radiation of a wave of constant amplitude, which indicates that the structure of the jump nevertheless exists, and then the amplitude of the radiated wave begins to oscillate chaotically with time, which indicates that the structure is unstable.

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